```
> restart : with(plots) : with(linalg) : with(DynamicSystems) :
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>
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I will begin with the equations t_1, t_2 and t_3. We do this in preperation to take the Jacobian matrix. Additionally, we will need a 3x3 identity matrix...

```
> t[1] := (λ - μ[x]·X - β·X·M);
  t[2] := (β·X·M - μ[y]·Y);
  t[3] := (r·μ[y]·Y - μ[m]·M - β·X·M);
  Identity := Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1]], 3, 3);
  T := Vector([t[1], t[2], t[3]], 3);
  vars := Vector([X, Y, M], 3);
```

$$t_1 := \lambda - \mu_x X - \beta X M$$

$$t_2 := \beta X M - \mu_y Y$$

$$t_3 := r \mu_y Y - \mu_m M - \beta X M$$

$$Identity := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T := \begin{bmatrix} \lambda - \mu_x X - \beta X M \\ \beta X M - \mu_y Y \\ r \mu_y Y - \mu_m M - \beta X M \end{bmatrix}$$

$$vars := \begin{bmatrix} X \\ Y \\ M \end{bmatrix} \tag{1}$$

Create the Jacobian with reference to our independent variables...

```
> J := jacobian(T, vars);
```

$$J := \begin{bmatrix} -\mu_x - \beta M & 0 & -\beta X \\ \beta M & -\mu_y & \beta X \\ -\beta M & r\,\mu_y & -\mu_m - \beta X \end{bmatrix} \tag{2}$$

Please note... The above matrix differs from what Ivana gave me, but only in the third row, first element. I'm not sure if she goofed or I did. But Maple returns a result that looks more symetrical, and Ivana's value of J ended up producing an intractible polynomial. Going forward, I will use Maple's version unless told that I am full of fail.

In our effort to find the Eigen values for the Jacobian, we must first take its determinant...

```
> determinant_jacobian := det(J - Identity * λ);
```

$$determinant_{linalg:-jacobian} := -\mu_x \mu_y \mu_m - \mu_x \mu_y \beta X - \mu_x \mu_y \lambda - \mu_x \lambda \mu_m - \mu_x \lambda \beta X - \mu_x \lambda^2 + \mu_x \beta X r \mu_y - \beta M \mu_y \mu_m - \beta M \mu_y \lambda - \beta M \lambda \mu_m - \beta M \lambda^2 - \lambda \mu_y \mu_m - \lambda \mu_y \beta X - \mu_y \lambda^2 - \lambda^2 \mu_m - \lambda^2 \beta X - \lambda^3 + \lambda \beta X r \mu_y \tag{3}$$

In order to proceed, we need to know the fixed point. This is a point in 3-space (because we have 3 independent variables) towards which the system will converge if the initial conditions and static parameters allow it. I looked at Ivana's algebra, and looked at the output of solve() for M, and chose Ivana's conclusion. For the sake of clarity in the following digression, I have reproduced the conclusion of her algebra below...

$$M\left(\frac{1}{\mu_x + \beta \cdot M} \cdot (r\beta\lambda - \beta\lambda) - \mu_m\right) = 0$$

Not knowing anything about the parameters, Maple gives a general solution for the M-component of the fixed point, and not only is it awful looking, but it unnecessarilly complicates the model. The only way for M to equal zero is for M to equal zero (haha). The static parameters of the model will not allow this component of the fixed point to be zero any other way. So we do the needful and assign the M-component of the fixed point to equal zero....

> $X := solve(t_1 = 0, X);\ Y := solve(t_2 = 0, Y);\ M := 0;$

$$X := \frac{\lambda}{\mu_x + \beta M}$$
$$Y := \frac{\beta \lambda M}{(\mu_x + \beta M)\, \mu_y}$$
$$M := 0 \tag{4}$$

Next we set the independent variables equal the fixed points and show that we indeed have a third-degree polynomial in λ....

> $determinant_{jacobian};$

$$-\mu_x \mu_y \mu_m - \mu_y \beta \lambda - \mu_x \mu_y \lambda - \mu_x \lambda \mu_m - \lambda^2 \beta - \mu_x \lambda^2 + \beta \lambda r \mu_y - \lambda \mu_y \mu_m - \frac{\lambda^2 \mu_y \beta}{\mu_x} - \mu_y \lambda^2 - \lambda^2 \mu_m - \frac{\lambda^3 \beta}{\mu_x} - \lambda^3 + \frac{\lambda^2 \beta r \mu_y}{\mu_x} \tag{5}$$

Now that we've verified that, we solve for λ, thereby executing Dr. Nagy's choice. Our results for the Eigen values will be given in terms of the constant parameters in the model....

> $results := solve(determinant_{jacobian} = 0, \lambda);$

$$results := -\mu_x, \frac{1}{2} \frac{1}{\beta + \mu_x} \Big(-\mu_y \beta - \mu_y \mu_x - \mu_m \mu_x + \beta r \mu_y + \big(\mu_y^2 \beta^2 + 2 \mu_y^2 \beta \mu_x - 2 \mu_y \beta \mu_m \mu_x - 2 \mu_y^2 \beta^2 r + \mu_y^2 \mu_x^2 - 2 \mu_y \mu_x^2 \mu_m - 2 \mu_y^2 \mu_x \beta r + \mu_m^2 \mu_x^2 - 2 \mu_m \mu_x \beta r \mu_y + \beta^2 r^2 \mu_y^2\big)^{1/2} \Big), -\frac{1}{2} \frac{1}{\beta + \mu_x} \Big(\mu_y \beta + \mu_y \mu_x + \mu_m \mu_x - \beta r \mu_y \tag{6}$$

$$+\left(\mu_y^2\,\beta^2+2\,\mu_y^2\,\beta\,\mu_x-2\,\mu_y\,\beta\,\mu_m\,\mu_x-2\,\mu_y^2\,\beta^2\,r+\mu_y^2\,\mu_x^2-2\,\mu_y\,\mu_x^2\,\mu_m-2\,\mu_y^2\,\mu_x\,\beta\,r+\mu_m^2\,\mu_x^2\right.$$
$$\left.-2\,\mu_m\,\mu_x\,\beta\,r\,\mu_y+\beta^2\,r^2\,\mu_y^2\right)^{1/2}\Big)$$

So we have two values for λ. Both are negative, assuming that all the parameters are positive.

Let us verify this, and execute Michael's request by resolving the values of λ into *actual numbers* by specifying the parameters that we have heretofore left indeterminant.

> $\mu_x := 1\cdot 10^{-6};\ \mu_y := 1.728\cdot 10^{-5};\ \mu_m := 1;\ r := 28;\ \mathit{evalf}(\mathit{results}[1]);\ \mathit{evalf}(\mathit{results}[2]);$

$$\mu_x := \frac{1}{1000000}$$
$$\mu_y := 0.00001728000000$$
$$\mu_m := 1$$
$$r := 28$$
$$-0.000001000000000$$
$$\frac{1}{\beta+0.000001000000000}\Big(0.5000000000\,\Big(0.0004665600000\,\beta-0.000001000017280$$
$$+\sqrt{2.176782336\;10^{-7}\,\beta^2-1.002256124\;10^{-9}\,\beta+9.999654403\;10^{-13}}\Big)\Big) \quad (7)$$

So there exist two Eigen values, both of which are negative.

One of these results doesn't include β. For the result that does, we can solve for it...

> $\mathit{solve}(\mathit{results}[2], \beta);$

$$-9.999999843\;10^{-7} \quad (8)$$

So β is equal (down to the limits of our precision in the parameters) to the first result. Both Eigen values appear to be essentially equal.

Just in case we need it, I will also ask Maple for the Jacobian determinant again. Now that the model's static parameters have been given concrete values, we will get a nice(r) quadratic...

> $0 = determinant_{jacobian};$

$$0 = -1.728000000\ 10^{-11} + 0.0004665600000\ \beta\ \lambda - 0.00001828001728\ \lambda + 465.5600000\ \lambda^2\ \beta - 1.000018280\ \lambda^2 - 1000000\ \lambda^3\ \beta - \lambda^3 \tag{9}$$

Thank you to everyone on the team whose hard work made it possible for me to do all this number-crunching during the free time in my freakish schedule.

---Ian

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